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Tubular Branes in Fluxbranes

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Abstract

We describe the construction of new configurations of self-gravitating p-branes with worldvolume geometries of the form $\mathbf{R}^{1,p-s} \times S^s$, with $1 \leq s \leq p$, *i.e., tubular branes*. Since such branes are typically unstable against collapse of the sphere, they must be held in equilibrium by a fluxbrane. We present solutions for string loops with non-singular horizons, as well as M5-branes intersecting over such loops. We also construct tubular branes which carry in their worldvolume a dissolved, lower dimensional brane (as in the dielectric effect), or an F-string. However, the connection between these solutions and related configurations that have been studied earlier in the absence of brane self-gravity, is unclear. It is argued that, at least in some instances, the self-gravitating solutions do not appear to be able to reproduce stable configurations of tubular branes.

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1 Introduction

One of the most important advances in string theory in the last years is the realization that D-branes admit dual descriptions, in terms of either open strings or closed strings. The open string description lends itself to quantum field theoretic analysis, whereas the closed string one finds its natural expression in terms of the different classical supergravities. Such a duality first lead to a succesful microscopic analysis of black holes [1], and, from there, to the celebrated AdS/CFT correspondence [2]. Nevertheless, it is probably fair to say that the closed string description lags, in many respects, behind the open string one.

To be more specific, consider the description of D-branes in terms of Dirac-Born-Infeld actions, which derives from the requirement of conformal invariance for open strings [3]. The Dirac-Born-Infeld picture allows one to easily consider some features of the spacetime dynamics of D-branes, as well as aspects of brane intersections and branes within branes. In some cases it provides a handle on processes beyond the reach of string perturbation theory, as for example, in the description of the decay of a RR field via the nucleation of the spherical D-branes that couple minimally to such a field [4]. It is also quite frequent that the dynamics of a D-brane in a certain curved background can only be analyzed by means of the "test-brane" approximation, where the gravitational backreaction of the brane is neglected.

Clearly, the difficulties in the closed string description come from the need to cope with a highly non-linear system, namely, general relativity (usually in high dimensions) typically coupled to a variety of scalar and p-form fields. Finding exact solutions to these field equations is usually a very complicated task, but it is nevertheless an imperative one, if the above mentioned duality is to be exploited in as complete a way as possible.

Our purpose in this paper is to report on some progress in this direction. We will present exact solutions to the equations of the low-energy effective action of string/M-theory, which describe p-branes whose worldvolume geometry is generically of the cylinder form $\mathbf{R}^{1,p-s} \times S^s$, *i.e., tubular branes.* It must be understood that the spherical part, S^s, does not wrap any non-trivial cycle of the spacetime. As a consequence, given that branes have a tension, their spherical part will tend to collapse down to a point. Such a collapse can be prevented by the introduction of an external field background, to which the brane couples. The simplest analogy is that of an electron-positron dipole held in (unstable) equilibrium by a carefully tuned electric field along the direction of the dipole. In the presence of gravity, such background fields concentrate under the influence of their self-gravity, and are referred to as *fluxbranes.* The best known instance among these is provided by the

Melvin universe [5]. Analyses of fluxbranes within the context of string theory include [6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

The study of gravitating spherical and tubular branes was initiated, in a beautiful piece of work, in [10], which we review in the next section. The context was that of Kaluza-Klein theories, or those that could be related to them via dualities. However, several brane configurations of interest are outside this range. For example, branes whose horizons are regular (instead of null singularities), cannot be constructed this way. Such branes are particularly important, since their cores provide regular geometries that are amenable to a detailed AdS/CFT description. The lowest dimensional instance, in the present context, would be the dipole formed by two Reissner-Nordstrom black holes with opposite charges, held in equilibrium by a background Melvin field. The description of such a *dihole*, including an embedding as an intersection of branes and antibranes, has been given in [16, 17]. In one dimension higher, there exists a five-dimensional string with a regular horizon: in this paper we will present a new exact solution that describes this kind of string in the shape of a loop (section 3). We also construct a configuration where three M5-branes (with different charges) intersect over one such loop (section 4). Unfortunately, the presumably most interesting configuration, a regular six-dimensional string loop, is beyond the approach herewith followed.

In a slightly different direction, given that D-branes can carry along their worldvolume field excitations that correspond to lower branes, or strings, dissolved in them (branes within branes), one can ask whether it might be possible to obtain solutions that describe tubular, or spherical, branes with a net charge[1]. This net charge would correspond to a lower dimensional brane, which, under the presence of the fluxbrane, has blown up into the tube, and dissolved itself in it. Such configurations were first analyzed, using the Dirac-Born-Infeld description, in [18]. More recently, they have been extensively studied within the context of the "dielectric effect" for D-branes [19]. Another system recently studied, where a tubular brane (called a "supertube" [20]) can support itself against collapse without the need of an external field, will also be of interest to us here. In section 5 we will present the construction of a number of supergravity solutions for D-brane tubes with this sort of charges. However, the analysis raises some doubts about whether these supergravity solutions adequately describe the supertubes of [20], or even the dielectric branes of [19]. It is unclear whether these self-gravitating solutions can describe tubular branes other than those in unstable

[1]Note that the spherical or tubular branes themselves, as the analogues of dipoles, do not carry a net charge.

equilibrium.

A remark on notation is in order. Spacetimes of different dimensionalities will abound, and a number of functions will be used, some of which will depend on the dimensionality of the space one is in. Then, when we employ the function defined below as $\Delta = r^2 - a^2 - \mu r^n$, we will be careful to specify which value of n is adequate in each situation. On the other hand, the function $\Sigma = r^2 - a^2\cos^2\theta$, will always be defined as this, whatever the dimension of the spacetime.

Finally, we note that when this work was in its last stages, a paper appeared in which spherical test branes in the presence of fluxbranes were studied [21]. Even more recently, another paper has appeared, which has more significant overlap with our work here [22]. In particular, the construction of the D4-brane blown up into a D6 in [22] is equivalent to our construction of the D0-brane blown up into a D2 sphere, in section 5 .

2 Spherical and Tubular Branes

In this section we describe, following [10], the construction of p-branes with worldvolume geometry $\mathbf{R}^{1,p-s} \times S^s$, in a spacetime of $p+4$ dimensions. We will also provide the explicit metrics and fields for the solutions. We shall begin with the case $p = s$, *i.e.,* spherical branes.

2.1 Spherical Branes

The starting point is the Euclidean continuation of the rotating black hole in $D = p+4$ dimensions, with rotation in only one plane [23]. To this, a flat time direction is added, resulting in a metric

$$\begin{aligned} ds^2_{D+1} &= -dt^2 + \frac{\Delta + a^2\sin^2\theta}{\Sigma}\left(dx - \frac{\mu a \sin^2\theta r^{5-D}}{\Delta + a^2\sin^2\theta}d\varphi\right)^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) \\ &+ \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\Omega^2_{D-4}\,, \end{aligned} \tag{1}$$

where

$$\begin{aligned} \Delta &= r^2 - a^2 - \mu r^{5-D}\,, \\ \Sigma &= r^2 - a^2\cos^2\theta\,. \end{aligned} \tag{2}$$

For $D \geq 5$, which will be the case of interest here, the polar coordinate θ runs along $0 \leq \theta \leq \pi/2$, while $\varphi \in [0, 2\pi]$ is an azimuthal angle. On the other hand, given the presence of a bolt (a "Euclidean horizon") at the largest root of Δ, $r = r_+$, then the coordinate r

takes only values $r \geq r_+$, and the coordinate x must be identified periodically, with a period equal to the inverse of the black hole temperature. Further details on the geometric aspects of these identifications can be found in[10]. Here we directly proceed to dimensionally reduce this solution along the direction of x. This results in a D-dimensional solution of a theory with action[2]

$$I = \frac{1}{16\pi} \int d^D x \left[R - \frac{1}{2}(\partial\phi)^2 - e^{\sqrt{\frac{2(D-1)}{D-2}}\phi} F^2 \right] . \tag{3}$$

The Einstein-frame metric of this solution is

$$\begin{aligned} ds^2_D &= \left(\frac{\Delta + a^2 \sin^2\theta}{\Sigma} \right)^{\frac{1}{D-2}} \left[-dt^2 + \Sigma \left(\frac{dr^2}{\Delta} + d\theta^2 \right) + r^2 \cos^2\theta d\Omega^2_{D-4} \right] \\ &+ \left(\frac{\Sigma}{\Delta + a^2 \sin^2\theta} \right)^{\frac{D-3}{D-2}} \Delta \sin^2\theta d\varphi^2 \, , \end{aligned} \tag{4}$$

while the Kaluza-Klein gauge potential and dilaton are

$$A_\varphi = -\frac{\mu a \sin^2\theta}{2r^{D-5}(\Delta + a^2 \sin^2\theta)} \, , \tag{5}$$

and

$$e^\phi = \left(\frac{\Delta + a^2 \sin^2\theta}{\Sigma} \right)^{\sqrt{\frac{D-1}{2(D-2)}}} . \tag{6}$$

A dual theory, where magnetic monopoles are mapped into $(D-4)$-branes that act as electrical sources for a $(D-2)$-form field strength, is obtained by tranforming

$$\tilde{\phi} = -\phi, \qquad F_{[D-2]} = 2 e^{\sqrt{\frac{2(D-1)}{D-2}}\phi} * F \, , \tag{7}$$

the metric remaining invariant. The dual theory is

$$I = \frac{1}{16\pi} \int d^5 x \left(R - \frac{1}{2}(\partial\tilde{\phi})^2 - \frac{1}{2(D-2)!} e^{\sqrt{\frac{2(D-1)}{D-2}}\tilde{\phi}} F^2_{[D-2]} \right) . \tag{8}$$

Then, the electric dual to (5) is

$$A_{t\psi_1 \ldots \psi_{D-4}} = \frac{\mu a \cos^{D-3}\theta \; \epsilon(\Omega_{D-4})}{\Sigma} \, , \tag{9}$$

where $\epsilon(\Omega_{D-4})$ is the volume factor for the $(D-4)$-sphere, parametrized by angular coordinates ψ_i. For this theory, the Kaluza-Klein interpretation is lost.

[2]Our conventions differ from [10] in the sign and normalization of the dilaton.

The reasoning used in [10] to argue that the solution (4), (5), (6), describes a spherical distribution of monopole charges was based on the topological structure of the higher dimensional solution. Here we follow a different, complementary approach, that works directly with the reduced solution. It is perhaps less elegant, but it can be applied as well to solutions that do not admit a Kaluza-Klein interpretation (*e.g.,* for other values of the dilaton coupling). For the case of $D = 4$, this approach has been used in [24] and [16], where a closely related detailed analysis can be found. The idea is to explicitly exhibit that on a sphere S^{D-4}, specified by $r = r_+$, with r_+ being the largest root of Δ, and by $\theta = 0$, there lie magnetic monopole charges. The charges are uniformly distributed over the sphere.

In order to study the solution near this sphere it is convenient to first perform the change of coordinates

$$\begin{aligned} r &= r_+ + \frac{\rho}{2}(1+\cos\bar\theta) \\ \sin^2\theta &= \frac{2\rho}{\Delta'_+}(1-\cos\bar\theta)\,, \end{aligned} \tag{10}$$

where

$$\Delta'_+ \equiv \frac{d\Delta}{dr}|_{r=r_+} = 2r_+ + (D-5)\mu r_+^{4-D}\,. \tag{11}$$

Then one takes ρ to be much smaller than any other length scale involved, so as to get close to the locus $(r = r_+, \theta = 0)$. In this way, one finds that the metric in the region close to this sphere approaches

$$\begin{aligned} ds_D^2 &= g(\bar\theta)^{\frac{1}{D-2}}\left[\left(\frac{\rho}{q}\right)^{\frac{1}{D-2}}(-dt^2 + r_+^2 d\Omega_{D-4}^2) + \left(\frac{\rho}{q}\right)^{-\frac{D-3}{D-2}}(d\rho^2+\rho^2 d\bar\theta^2)\right] \\ &+ g(\bar\theta)^{-\frac{D-3}{D-2}}\left(\frac{\rho}{q}\right)^{-\frac{D-3}{D-2}}\rho^2\sin^2\bar\theta d\varphi^2\,, \end{aligned} \tag{12}$$

where we have defined $q \equiv \frac{r_+^2 - a^2}{\Delta'_+}$. The dilaton and gauge potential approach

$$\begin{aligned} e^\phi &= \left(\frac{\rho g(\bar\theta)}{q}\right)^{\sqrt{\frac{D-1}{2(D-2)}}}\,, \\ A_\varphi &= -\frac{a}{\Delta'_+}\frac{q(1-\cos\bar\theta)}{g(\bar\theta)}\,. \end{aligned} \tag{13}$$

If the factor $g(\theta)$ were replaced by 1, this would be precisely the solution near the core of a set of Kaluza-Klein magnetic monopoles, uniformly distributed over the sphere S^{D-4} [3]. The

[3] Since $\rho \ll r_+$, near the core this sphere looks very large, and nearly flat. Effectively, $r_+^2 d\Omega_{D-4}^2 \to \sum_{i=1}^{D-4} dx_i^2$.

configuration, though, is angularly distorted by the presence of

$$g(\bar{\theta}) = \frac{1}{2}\left[1 + \cos\bar{\theta} + \left(\frac{2a}{\Delta'_+}\right)^2 (1 - \cos\bar{\theta})\right] . \tag{14}$$

This distortion is also related to another feature of the solution (4), namely, the presence of conical singularities. To see these, observe that $r = r_+$ is a fixed-point set of the generator of rotations ∂_φ, hence a part of its axis. The conical singularity is then evidenced by the fact that the ratio of circumference to proper radius,

$$\lim_{r \to r_+} \frac{2\pi}{\sqrt{g_{rr}}} \frac{d\sqrt{g_{\varphi\varphi}}}{dr} = \frac{\pi \Delta'_+}{a} , \tag{15}$$

does not equal the canonical value 2π if $\mu \neq 0$. As a matter of fact, such conical singularities are typical of situations where there are unbalanced forces. In the present case, this is indeed expected, since the monopoles attract each other, both by magnetic and gravitational forces[4].

The sphere S^{D-4} and its interior lie at $r = r_+$. In the interior, θ, or more precisely, $\cos\theta$, plays the role of the radial coordinate inside the sphere: notice that, at fixed $r = r_+$, the "volume radius" of the sphere is proportional to $\cos\theta$. In particular, the center of the sphere is at $\theta = \pi/2$, while the surface, where the monopoles lie, is at $\theta = 0$. The conical singularity extends over all of the interior, but apart from this, the interior of the sphere presents no other singularities.

Let us also remark that, if instead of considering the limit of small ρ above, we had sent $a \to \infty$, while keeping $r - r_+$, $a\sin^2\theta$, and q finite, then we would have recovered precisely the solution for a planar (infinite) distribution of magnetic monopoles. Hence we see that the parameter a controls, through r_+, the size of the monopole sphere.

In order to balance the system, and cancel the conical singularities, one can introduce an external magnetic field. The way to introduce a magnetic field fluxbrane in a KK setup involves twisting the compactification direction [26]. Effectively, one shifts the coordinate $\varphi \to \varphi - Bx$, and reidentifies points appropriately. Then, upon reduction, the solution becomes

$$ds_D^2 = \Lambda^{\frac{1}{D-2}}\left[-dt^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + r^2\cos^2\theta d\Omega_{D-4}^2\right] + \frac{\Delta\sin^2\theta}{\Lambda^{\frac{D-3}{D-2}}} d\varphi^2 , \tag{16}$$

where

$$\Lambda = \frac{\Delta + a^2\sin^2\theta + 2Ba\mu r^{5-D}\sin^2\theta + B^2\sin^2\theta[(r^2-a^2)^2 + \Delta a^2\sin^2\theta]}{\Sigma} . \tag{17}$$

[4]Bear in mind that monopoles at antipodal points on the sphere have opposite charges, since for $D > 4$ the sign of the magnetic charge for a two-form field strength depends on a choice of orientation [10]. This point is perhaps more obvious for the electric brane solutions.

The gauge potential is now

$$A_\varphi \;=\; -\frac{\mu a r^{5-D} + B[(r^2-a^2)^2 + \Delta a^2 \sin^2\theta]}{2\Lambda\Sigma}\sin^2\theta\,, \tag{18}$$

and $e^\phi = \Lambda^{\sqrt{\frac{D-1}{2(D-2)}}}$.

Let us now see whether we can tune the field B in such a way that the conical singularities are removed. With the B field on, the circumference/radius ratio becomes, as one approaches the portion of the axis along $r = r_+$,

$$\lim_{r\to r_+}\frac{2\pi}{\sqrt{g_{rr}}}\frac{d\sqrt{g_{\varphi\varphi}}}{dr} = \frac{\pi\Delta'_+}{a + B(r_+^2 - a^2)}\,. \tag{19}$$

Hence, the axis will be regular if we choose

$$B = \frac{\Delta'_+ - 2a}{2(r_+^2 - a^2)} = \frac{(D-3)r_+ + (D-5)a}{2r_+(r_+ + a)}\,. \tag{20}$$

Analyze now the solution in the region near $(r = r_+, \theta = 0)$, by means of the same transformation (10). One finds the metric becomes exactly like (12), but now with

$$g(\bar\theta) = \frac{1}{2}\left[1 + \cos\bar\theta + 4\left(\frac{a + B(r_+^2 - a^2)}{\Delta'_+}\right)^2(1 - \cos\bar\theta)\right]\,. \tag{21}$$

Therefore, if the field is tuned to the value (20), we find $g(\bar\theta) = 1$, and the angular distortion of the core disappears completely. The solution approaches exactly the core of the monopole. On the other hand, the gauge potential near the core,

$$A_\varphi = -\frac{a + B(r_+^2 - a^2)}{\Delta'_+ g(\bar\theta)} q(1 - \cos\bar\theta)\,, \tag{22}$$

recovers its precise spherical monopolar form for the equilibrium value for B, (20).

The way we have presented our results will be the one we shall follow throughout the paper. Namely, in order to show that a configuration describes a spherical brane, it suffices to consider first the simpler situation where the background fluxbrane is absent. The solution near the core is slightly distorted, and presents a conical singularity, but nevertheless it already possesses the main features of the spherical brane. Having identified the configuration properly, it is a straightforward matter to balance the system by immersing it in the field of a fluxbrane.

2.2 Tubular Branes

This construction can be modified very easily to construct tubular p-branes with worldvolume geometry $\mathbf{R}^{1,p-s} \times S^s$. To this effect, since we are considering an s-dimensional sphere, first set $D = s + 4$ in the metric (1). Then, simply add a number $p - s$ of flat dimensions along with the time coordinate. Reduction along the x coordinate results into the required tubular p-brane. The solution lives in a spacetime of $p + 4$ dimensions.

In particular, for ten-dimensional string theory, one obtains in this way a whole range of tubular D6-branes, with worldvolume geometry $\mathbf{R}^{1,6-s} \times S^s$. The case of $s = 0$ is the D6-$\bar{\text{D}}$6 state of [24]. T-duality can be applied along some or all of the $6 - s$ flat worldvolume directions, and results into delocalized (smeared) configurations of other tubular Dp-branes, with $p \leq 6$. In conjunction with S-duality, one can as well construct delocalized configurations for fundamental strings (as noted in [10]) and solitonic fivebranes. Some particular instances of this construction can be found in [25].

3 Other dilaton couplings, and non-singular string loops

There are several shortcomings to the construction described in the previous section. Notice that, if the brane is to be a localized one, then the dimension of the spacetime it lives in is restricted to the value $p + 4$. It appears to be very difficult to overcome this restriction, and in fact we will have nothing to add in this respect. Another constraint arises on the allowed values of the dilaton coupling. These are restricted to those available by Kaluza-Klein reduction. In particular this means that the above solutions cannot be used to describe large classes of intersecting branes. Among the brane intersections, a particularly interesting class is constituted by those that intersect over a regular horizon (even though each individual, delocalized brane may have a null singular core). Examples include the Reissner-Nordstrom black hole in four dimensions, which can be embedded as, *e.g.,* an intersection of four D3-branes, and the five-dimensional and six-dimensional black strings, which appear at a triple intersection of M5-branes [27]. Now, the question is whether one can construct configurations where the branes intersect over a loop or a sphere. The lowest dimensional example is that where, instead of a loop or a sphere, the branes intersect over a dipole formed by two black holes in four dimensions, with equal masses and opposite charges—a dihole. The dihole solution formed by the intersection of two sets of four D3-branes each, has been already constructed in [17]. Going to a one-dimensional intersection, the triple intersection of M5-branes over a string loop will be built in the next section. Here, as a first step, we build string loops in five dimensions with arbitrary dilaton coupling α. In the particular case of $\alpha = 0$, where the dilaton decouples, the string loop will have a non-singular horizon. It would also be interesting to build a six-dimensional loop of string with a regular horizon. However, this requires a solution outside the class of spherical p-branes in $p+4$ dimensions (instead, $p+5$ would be needed), and therefore it is beyond the techniques employed here.

Hence, let us consider the theories and solutions in the previous section, for the case $D = 5$. They are a particular case of dilatonic theories

$$I = \frac{1}{16\pi}\int d^5x\, \left(R - \frac{1}{2}(\partial\phi)^2 - e^{\alpha\phi}F^2\right)\,, \tag{23}$$

namely, the ones with $\alpha = 2\sqrt{2/3}$. For a given value of α, it is convenient to define a parameter N via

$$\alpha^2 = \frac{4}{N} - \frac{4}{3}\,. \tag{24}$$

When the solutions are realized as brane intersections, N is actually the number of intersecting branes. The case of Kaluza-Klein theory corresponds to $N = 1$, and the non-dilatonic theory is obtained for $N = 3$. The dual theory, where magnetic monopoles are mapped into electric strings, is obtained by changing to $\tilde{\phi} = \phi$ and $H = 2e^{\alpha\phi} * F$,

$$I = \frac{1}{16\pi}\int d^5x\, \left(R - \frac{1}{2}(\partial\phi)^2 - \frac{1}{12}e^{\alpha\tilde{\phi}}H^2\right)\,. \tag{25}$$

We have found a solution to these theories for arbitrary α, *i.e.,* arbitrary $N \leq 3$, which generalizes the Kaluza-Klein loop of monopoles that was described in the previous section. The metric is

$$\begin{aligned} ds^2 &= \left(\frac{\Delta + a^2\sin^2\theta}{\Sigma}\right)^{N/3}\left[-dt^2 + r^2\cos^2\theta d\psi^2 + \frac{\Sigma^N}{[\Delta + (\mu + a^2)\sin^2\theta]^{N-1}}\left(\frac{dr^2}{\Delta} + d\theta^2\right)\right] \\ &+ \left(\frac{\Sigma}{\Delta + a^2\sin^2\theta}\right)^{2N/3}\Delta\sin^2\theta d\varphi^2\,, \end{aligned} \tag{26}$$

and the dilaton and magnetic field

$$\begin{aligned} e^{\phi} &= \left(\frac{\Delta + a^2\sin^2\theta}{\Sigma}\right)^{N\alpha/2} \\ A_\varphi &= -\frac{\sqrt{N}}{2}\frac{\mu a\sin^2\theta}{\Delta + a^2\sin^2\theta}\,. \end{aligned} \tag{27}$$

The electric dual strings have 2-form potential ($H = dB$)

$$B_{t\psi} = -\sqrt{N}\frac{\mu a\cos^2\theta}{\Sigma}\,. \tag{28}$$

In this case, $\Delta = r^2 - a^2 - \mu$, hence $r_+ = \sqrt{\mu + a^2}$. Most aspects of this solution involve only a slight change from the Kaluza-Klein solution $N = 1$, except for one: the metric coefficients g_{rr} and $g_{\theta\theta}$ have acquired a new term, that is absent for $N = 1$. Actually, this new factor is analogous to the one that appears in the four dimensional diholes [28, 29, 16].

In the same fashion as described in the previous section, one sees that the string loop is located at $r = r_+ = \sqrt{\mu + a^2}$ and $\theta = 0$. The interior of the loop is the two-dimensional disk at $r = r_+$. On it, one moves from the boundary to the center by varying $\theta \in [0, \pi/2]$, and in the angular direction by varying $\psi \in [0, 2\pi]$. The transformation (10), in the limit of small ρ, reproduces the (distorted) geometry near the core of five-dimensional magnetic monopoles distributed along a circle, or alternatively, a five-dimensional electric string loop.

Again, these configurations present the expected conical singularities, which will be removed by the addition of a fluxbrane background. The Kaluza-Klein procedure of twisting the direction of reduction cannot be applied to these cases. However, a solution-generating transformation can be found for the theories (23), that extends to five dimensions the Harrison transformations of [26]. Given an axisymmetric solution to (23), where the only non-zero component of the gauge field is A_φ, and the metric $g_{ij}, g_{\varphi\varphi}$, $(i, j \neq \varphi)$, a new solution is generated by

$$\begin{aligned} g'_{ij} &= \lambda^{N/3} g_{ij} \qquad g'_{\varphi\varphi} = \lambda^{-2N/3} g_{\varphi\varphi} \\ e^{\phi'} &= e^{\phi}\lambda^{N\alpha/2} \qquad A'_\varphi = \frac{N}{2B\lambda}\left(1 + \frac{2}{N}BA_\varphi\right) + k\,, \end{aligned} \tag{29}$$

where

$$\lambda = \left(1 + \frac{2}{N}BA_\varphi\right)^2 + \frac{1}{N}B^2 g_{\varphi\varphi}e^{-\alpha\phi}\,, \tag{30}$$

and k is a constant that can be adjusted to fix Dirac strings: we will set $k = -N/(2B)$. Such a transformation generates a magnetic fluxbrane background, with the field strength along the axis asymptotically given by the parameter B. By dualization, one obtains an electric fluxbrane background associated to the three-form H.

It is straightforward to apply this transformation to the monopole string loops (31), and then dualize to obtain an electric string loop. Doing so, we obtain

$$\begin{aligned} ds^2 &= \Lambda^{N/3}\left[-dt^2 + r^2\cos^2\theta d\psi^2 + \frac{\Sigma^N}{[\Delta + (\mu + a^2)\sin^2\theta]^{N-1}}\left(\frac{dr^2}{\Delta} + d\theta^2\right)\right] \\ &+ \Lambda^{-2N/3}\Delta\sin^2\theta d\varphi^2\,, \end{aligned} \tag{31}$$

and $e^{\phi} = \Lambda^{N\alpha/2}$, with

$$\Lambda = \frac{\Delta + a^2\sin^2\theta + \frac{2}{\sqrt{N}}B\mu a\sin^2\theta + \frac{1}{N}B^2\sin^2\theta[(r^2 - a^2)^2 + \Delta a^2\sin^2\theta]}{\Sigma}\,, \tag{32}$$

and

$$B_{t\psi} = -\sqrt{N}\left[\frac{\mu a\cos^2\theta}{\Sigma}\left(1-\frac{aB}{\sqrt{N}}\sin^2\theta\right)\left(1+(1-2/\sqrt{N})\frac{aB}{\sqrt{N}}\sin^2\theta\right)-\frac{1}{N}Br^2\cos^2\theta\right]. \tag{33}$$

In particular, the last term in the potential corresponds to the asymptotic electric fluxbrane.

At this point, we can tune the fluxbrane strength B so as to balance the system and cancel the conical singularities on $r = r_+$. The field that achieves this is

$$B = \sqrt{N}\frac{\sqrt{\mu + a^2} - a}{\mu}. \tag{34}$$

A choice of sign for $\sqrt{\mu + a^2}$ has been made here. As in [16], we have chosen the one that yields $B \to 0$ as $a \to \infty$.

For this value of the field, the solution near the loop becomes precisely

$$ds^2 = \left(\frac{\rho}{q}\right)^{N/3}(-dt^2 + r_+^2 d\psi^2) + \left(\frac{\rho}{q}\right)^{-2N/3}[d\rho^2 + \rho^2(d\bar{\theta}^2 + \sin^2\bar{\theta}d\phi^2)], \tag{35}$$

($q = \mu/(2\sqrt{\mu + a^2})$) *i.e.,* , the distortion has disappeared and the solution takes precisely the form of the core of the dilatonic five-dimensional string. Observe that, for the case $N = 3$, the surface $\rho = 0$ is a regular horizon, in fact in this case the geometry is an $AdS_3 \times S^2$ throat, where the coordinate ψ is periodic. [5] The horizon has zero area.

An interesting possibility would be to add momentum running along the loop. For straight strings this is achieved by boosting the solution along the direction of the string, but this cannot be done in the present case. If such a solution were constructed, one could envisage balancing the system without the need of an external field: the centrifugal force caused by the rotation of the string might compensate for the tendency of the loop to collapse. Also, the area of the horizon might be nonvanishing for this configuration.

Finally, we want to mention that despite the ease with which we have generalized to arbitrary dilaton coupling the solutions for string loops, it appears much more difficult to do the same for the solutions for spherical p-branes with $p \geq 2$. Also, non-extremal string loops (with non-degenerate horizons) would be of interest. For the case in one dimension less, *i.e.,* the dihole solutions, it has been found that the complication of the metrics increases enormously when considering the non-extremal versions [31], so perhaps the non-extremal string loops will be equally complicated.

[5] Our analysis is a local one. For related global issues, see [30].

4 M5-branes intersecting over a string loop

Having described how to build string loops for arbitrary N (*i.e.,* arbitrary dilaton coupling α), we now turn to see how the solutions for integer values of N, namely $N = 1, 2, 3$ can be obtained at the intersection of N branes. In particular, we will consider the intersection of three M5-branes over a string [27].

In these solutions, the charges of each of the M5-branes will be independent parameters. Hence, the solution will have three independent gauge field components. It was already found in [17] that, in four dimensions, the step from the dilatonic dihole solutions to the multicomponent solutions is not too complicated. It works as well here. We have found the solution for three M5-branes intersecting over a string loop explicitly as the metric

$$\begin{aligned} ds^2 &= (T_1T_2T_3)^{1/3}\left[-dt^2 + r^2\cos^2\theta d\psi^2 + \frac{\Sigma_1\Sigma_2\Sigma_3}{(\Delta+\gamma^2\sin^2\theta)^2}\left(\frac{dr^2}{\Delta}+d\theta^2\right)\right] + \frac{\Delta\sin^2\theta}{(T_1T_2T_3)^{2/3}}d\varphi^2 \\ &+ \frac{(T_2T_3)^{1/3}}{T_1^{2/3}}(dy_1^2+dy_2^2) + \frac{(T_1T_3)^{1/3}}{T_2^{2/3}}(dy_3^2+dy_4^2) + \frac{(T_1T_2)^{1/3}}{T_3^{2/3}}(dy_5^2+dy_6^2)\,, \end{aligned} \tag{36}$$

and four-form field strength,

$$F_{[4]} = 3(dA_{(1)}\wedge dy_1\wedge dy_2 + dA_{(2)}\wedge dy_3\wedge dy_4 + dA_{(3)}\wedge dy_5\wedge dy_6)\,, \tag{37}$$

with

$$A_{(i)} = -\frac{\mu_i a_i\sin^2\theta}{\Delta + a_i^2\sin^2\theta}d\varphi\,. \tag{38}$$

The solution depends on four parameters, which correspond to the three charges of the M5-branes, and the size of the string loop where they intersect. We have chosen the parameters to be a_i ($i = 1, 2, 3$) and γ. Other, non-independent parameters, are defined as $\mu_i = \gamma^2 - a_i^2$. In terms of these, the functions above are

$$\begin{aligned} \Delta &= r^2 - \gamma^2\,, \\ \Sigma_i &= r^2 - a_i^2\cos^2\theta\,, \\ T_i &= \frac{\Delta + a_i^2\sin^2\theta}{\Sigma_i}\,. \end{aligned} \tag{39}$$

In the case that the three a_i are all equal, one recovers the solution (31) for $N = 3$. If two (one) a_i are equal, and the other one (two) vanishes, then the solutions for $N = 2$ ($N = 1$) are recovered.

All three M5-branes share the direction ψ, *i.e.,* the string loop. They intersect pairwise over a tubular three-brane, $\mathbf{R}^{1,2}\times S^1$, with spatial coordinates y_i, y_j, ψ.

As in the cases previously studied, the solution needs to be balanced by a fluxbrane. Fluxbrane backgrounds can be introduced here by means of a straightforward generalization of the Harrison transformation for multiple gauge fields developed in [11]. The situation is similar to the one studied in [17] for quadruple intersections of branes on diholes, so we will only recapitulate the main features. One can turn on three different fluxbranes, each along the direction of either of the three M5-branes. However, it is not necessary to turn on all three fluxbranes in order to balance the system. Instead, the conical singularity in the metric can be removed by turning on a single fluxbrane, which pulls on only one of the M5-branes. This single force can be sufficient to equilibrate the attraction. When the system is equilibrated in this fashion, the geometry of the string loop at the intersection is completely non-singular. However, a certain amount of deformation remains, in the sense that even if the geometry approximates $AdS_3 \times S^2$, the curvature of each of the two pieces is not constant. Only when all three intersecting fluxbranes are turned on, and tuned separately, does the distortion of the horizon disappear completely. For more details, we refer the reader to the detailed study in [17].

5 Blown-up strings and branes

The spherical and tubular branes we have been considering this far carry no net charge. In this sense, they are the higher dimensional analogues of electron-positron dipoles. A D-brane, however, can carry along its worldvolume a gauge field that corresponds to a lower-dimensional brane that is dissolved in it. Hence, one could envisage situations where the brane spheres and tubes of the previous sections carry a net charge, corresponding to some brane that is dissolved in them.

One such situation was considered already some time ago in [18]. It is known that an F-string can be dissolved in the worldvolume of a D-brane, in the form of an electric worldvolume field. Hence, it was pointed out in [18] that the F-string could alternatively be viewed as a collapsed Dp-brane tube $\mathbf{R}^{1,1} \times S^{p-1}$, the Dp-brane carrying an electric field along the straight brane direction, and the remaining $p-1$ directions being a sphere collapsed to zero size. It was further observed that this collapse could be prevented by applying a uniform RR $p+2$-form field strength, to which the Dp-brane couples. The collapsed brane (*i.e.,* the F-string) would be blown up into the $\mathbf{R}^{1,1} \times S^{p-1}$ tubular brane, and an equilibrium configuration should be possible. The case of $p = 2$ was worked out in detail in [18], where it was also pointed out that a similar effect would be expected for higher p, as well as for the cases in which a D-brane can be dissolved inside a higher D-brane. It must be pointed out that the configurations envisaged in [18] were in unstable equilibrium.

Actually, such configurations of Dp-branes blown up into D$(p+2n)$-branes in the presence of a uniform $p+2n+2$-form field strength were studied in greater detail, and from other perspectives, in [19], and it was remarked that stable configurations would also be possible. In analogy with electric dipoles, the D-branes behave as dielectrics. The case of a D0-brane blown up into a spherical D2-brane is perhaps the simplest, paradigmatic instance of such a phenomenon.

More recently, a mixture between these two possibilities, with remarkable features of itself, has been devised in [20]. The system considered consists of a D2-brane tube which carries along its worldvolume crossed electric and magnetic fields. These correspond to F-strings and D0-branes, respectively, dissolved in the tube. It turns out that the angular momentum of this electromagnetic field can balance the tube tension at a finite radius, in a stable configuration, without the need of an applied external field. Moreover, such a tube has been shown to be supersymmetric.

What is missing in all these studies is the introduction of consistent self-gravity into the systems. It must be mentioned that approximate solutions for D3-branes blowing up into D5-branes were built in [32]. Our aim here is to include self-gravity in an exact fashion.

Our approach will be based on the following reasoning. The starting point in the construction of tubular Dp-branes was the Euclidean continuation of the higher-dimensional rotating black hole. We are now interested in tubular branes which carry a net charge, under a field different to the one they minimally couple to. Since all brane charges can be generated, via dimensional reduction and then via dualities, from the charges of the M5 and M2 in eleven dimensions, it appears that the configurations we seek should be obtainable from the Euclidean continuation of the solutions for the rotating M5 and M2 brane, and possibly their intersections too. Fortunately, such solutions have been constructed in [33]. We will see that the expectation that such a procedure may work is borne out in practice.[6]

In this section we shall describe configurations corresponding to the three sorts of situations described above: (a) an F-string blown up into a tubular Dp-brane. (b) a D0-brane blown up into a spherical D2-brane. (c) an F-string and a D0-brane blown up into a tubular D2-brane. For the same reasons as explained in Section (2), these configurations, when considered in ten dimensions, will only be localized along four of the directions transverse

[6] A construction along similar lines was considered in [25], but the interpretation of the solutions is missing there.

to the tubular or spherical brane. Hence, generically, the branes will be delocalized along a number of directions.

5.1 The F1 blown up into a tubular Dp

For definiteness, we will start with the situation where the F-string blows up into a D6-brane of worldvolume $\mathbf{R}^{1,1} \times S^5$. To construct this configuration, we start with a solution in eleven dimension describing a rotating M2-brane [33]. After Wick-rotating the time coordinate, as well as one of the brane coordinates, one has

$$\begin{aligned} ds_{11}^2 &= H^{-2/3}\left[-dt^2 + dz^2 + \frac{\Delta + a^2\sin^2\theta}{\Sigma}\left(dx_{11} - \frac{\mu a\cosh\delta\sin^2\theta}{r^4(\Delta + a^2\sin^2\theta)}d\varphi\right)^2\right] \\ &+ H^{1/3}\left[\Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\Omega_5^2\right], \end{aligned} \tag{40}$$

where

$$\begin{aligned} \Delta &= r^2 - a^2 - \frac{\mu}{r^4}, \\ H &= 1 + \frac{\mu\sinh^2\delta}{r^4\Sigma}, \end{aligned} \tag{41}$$

and the three-form potentials

$$\begin{aligned} B_{tzx_{11}} &= \frac{\mu\cosh\delta\,\sinh\delta}{r^4\Sigma H}, \\ B_{tz\varphi} &= -\frac{a\mu\sinh\delta\,\sin^2\theta}{r^4\Sigma H}. \end{aligned} \tag{42}$$

The M2-brane is extended along the directions z and x_{11}. In the situation where the brane is actually rotating, x_{11} would be the time direction, while t would be a spatial direction along the brane, but we have Wick-rotated both of them, as well as the rotation parameter a. In this way we are in a situation similar to that in Section 2, but now a net M2-brane charge is present. The parameter δ is associated to this charge. In the limit where $\mu \to 0$ and $\delta \to \infty$, keeping $\mu e^{2\delta}$ finite, one recovers the solution for an M2-brane, in prolate spheroidal coordinates.

The solution can now be reduced down to ten dimensions along the direction of x_{11}, to a solution of type IIA string theory. Notice that the string coupling constant, which is determined by the asymptotic radius of the compact circle, is fixed by the periodicity requirements for x_{11}. In the process, the M2-brane becomes a F-string along the z direction.

But, as expected from the analysis in the previous sections, we also generate a tubular D6-brane, with worldvolume geometry $\mathbf{R}^{1,1} \times S^5$. Explicitly, the string metric for the solution is

$$\begin{aligned} ds^2 = & \left(\frac{\Delta + a^2 \sin^2\theta}{\Sigma}\right)^{1/2} \left[H^{-1}(-dt^2 + dz^2) + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right)\right. \\ & \left. + r^2\cos^2\theta d\Omega_5^2 + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta} d\varphi^2\right], \end{aligned} \tag{43}$$

with dilaton

$$e^\phi = H^{-1/2}\left(\frac{\Delta + a^2\sin^2\theta}{\Sigma}\right)^{3/4}, \tag{44}$$

and potentials

$$\begin{aligned} A_\varphi &= -\frac{\mu a \cosh\delta\ \sin^2\theta}{r^4(\Delta + a^2\sin^2\theta)}, \\ B_{tz} &= \frac{\mu\cosh\delta\ \sinh\delta}{r^4\Sigma H}, \\ B_{tz\varphi} &= -\frac{a\mu\sinh\delta\ \sin^2\theta}{r^4\Sigma H}. \end{aligned} \tag{45}$$

A_φ is created by the D6-brane tube, while B_{tz} is the Kalb-Ramond potential for the F-string. The remaining $B_{tz\varphi}$ appears as a consequence of the mixing between the latter two in the equations of motion, and vanishes whenever only one of the two sorts of charges is present.

One may question whether the F-string has expanded along with the D6-brane, or instead it remains back at the center of the sphere S^5. The correct answer is actually the former. First, note that if the F-string were inside the sphere, the core of the string should appear as a singularity at $r = r_+$, and at some $\theta \in [0, \pi/2]$ (recall that $\theta = \pi/2$ corresponds to the center of the sphere, and $\theta = 0$ to the boundary). But, as a matter of fact, at $r = r_+$, and any value of θ, the function H remains finite. It follows from a trivial extension of our analysis in Section 2 that, in this case, the only singularity present inside the sphere is the expected conical defect.

The evidence that the F-string is actually dissolved in the D6-brane tube, at $r = r_+$ and $\theta = 0$, is borne out from the study of the solution near the tube. Performing the change of coordinates (10), and going near the core (*i.e.*, to small ρ), results into $H \to \cosh^2\delta$ and

$$\begin{aligned} ds^2 &= \left(\frac{g(\bar\theta)\rho}{q}\right)^{1/2}\left[\frac{-dt^2 + dz^2}{\cosh^2\delta} + r_+^2 d\Omega_5^2 + \frac{q}{\rho}(d\rho^2 + \rho^2 d\bar\theta^2)\right] \\ &+ \left(\frac{q}{g(\bar\theta)\rho}\right)^{1/2}\rho^2\sin^2\bar\theta d\varphi^2, \end{aligned} \tag{46}$$

where $g(\bar{\theta})$ is as in (14), and $q = (r_+^2 - a^2)/\Delta'_+$ is the same as was defined after (12).

This should reproduce the solution for an F-string dissolved into a flat D6-brane, up to the angular distortion factors, which are due to unbalanced forces, and the replacement of the 5-sphere by a flat $\mathbf{R}^5$. The string-frame metric for such a bound state was constructed in [35], as

$$ds^2 = \tilde{f}^{1/2}[f^{-1}(-dt^2 + dz^2) + \tilde{f}^{-1}d\mathbf{x}_{(5)}^2 + d\rho^2 + \rho^2 d\Omega_2^2]\,, \tag{47}$$

with

$$f = 1 + \frac{k}{\rho}\,, \qquad \tilde{f} = 1 + \frac{k\sin^2\alpha}{\rho}\,. \tag{48}$$

The D6-brane is recovered for $\alpha = \pi/2$, and the delocalized F-string appears for $\alpha = 0$. The core limit is $\rho \to 0$, in which the metric becomes

$$ds^2 = \left(\frac{\rho}{k\sin^2\alpha}\right)^{1/2}[\sin^2\alpha(-dt^2 + dz^2) + d\mathbf{x}_{(5)}^2] + \left(\frac{\rho}{k\sin^2\alpha}\right)^{-1/2}(d\rho^2 + \rho^2 d\Omega_2^2)\,. \tag{49}$$

Notice that the singular behavior near the core is dominated by the D6-brane. Comparing to (46), we can identify the solutions, up to the distortion factors, by making $\cosh\delta = 1/\sin\alpha$, and $q = k\sin^2\alpha$. Although we have not written them explicitly, one can easily check that the one-, two-, and three-form potentials also map correctly between both configurations, up to the distortion introduced by $g(\bar{\theta})$.

The way to proceed now should be clear from what we have seen in the previous sections. A RR fluxbrane background that couples to the D6-brane can be introduced, via a twist in the reduction from eleven to ten dimensions. One then finds the string-frame metric

$$ds^2 = \Lambda^{1/2}\left[H^{-1}(-dt^2 + dz^2) + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + r^2\cos^2\theta d\Omega_5^2 + \frac{\Delta\sin^2\theta}{\Lambda}d\varphi^2\right]\,, \tag{50}$$

and dilaton $e^\phi = H^{-1/2}\Lambda^{3/4}$, with

$$\begin{aligned}\Lambda \;&=\; \frac{\Delta + a^2\sin^2\theta + 2B\mu r^{-4}a\cosh\delta\sin^2\theta + B^2\sin^2\theta[(r^2-a^2)^2 + \Delta a^2\sin^2\theta]}{\Sigma} \\ &+\; \frac{B^2a^2\mu^2r^{-4}\sinh^2\delta\sin^2\theta}{\Sigma(\Delta + a^2\sin^2\theta)}\,.\end{aligned} \tag{51}$$

This fluxbrane exerts a pressure on the D6-brane tube, and the strength of the fluxbrane can be tuned so as to balance the system, *i.e.,* cancel the conical singularities at $r = r_+$. This happens for

$$B = \frac{6r_+ + 4a}{2r_+(r_+ + a)\cosh\delta} \tag{52}$$

(c.f. eq. (20)). For this value of the fluxbrane strength, the angular distortion of the core disappears, $g(\bar{\theta}) = 1$, and we recover precisely (49), with $r_+^2 d\Omega_5^2$ instead of $d\mathbf{x}_{(5)}^2$.

In order to build F-strings blown up into other tubular Dp-branes, with worldvolume $\mathbf{R}^{1,1} \times S^{p-1}$ (and delocalized along $6-p$ directions), consider the eleven-dimensional metric

$$
\begin{aligned}
ds_{11}^2 &= H^{-2/3}\left[-dt^2 + dz^2 + \frac{\Delta + a^2\sin^2\theta}{\Sigma}\left(dx_{11} - \frac{\mu a \cosh\delta \sin^2\theta}{r^{p-2}(\Delta + a^2\sin^2\theta)}d\varphi\right)^2\right] \\
&+ H^{1/3}\left[d\mathbf{x}_{(6-p)}^2 + \Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\Omega_{p-1}^2\right], \quad (53)
\end{aligned}
$$

where now $\Delta = r^2 - a^2 - \mu r^{2-p}$, $H = 1 + \frac{\mu \sinh^2\delta}{r^{p-2}\Sigma}$, and the three-form potentials are as in (42), but with r^{p-2} in the denominators, instead of r^4.

In a manner entirely analogous to the one just discussed, when reduced down to a solution of ten-dimensional type IIA supergravity, this results into an F-string spread (dissolved) along the worldvolume $\mathbf{R}^{1,7-p} \times S^{p-1}$ of a D6-brane. Then, T-duality along the straight $7-p$ directions yields the desired solution. The way to proceed is straightforward, and therefore we shall not give any further details.

5.2 The D0 blown up into a D2 sphere

The construction is very similar to the one in the previous subsection, but this time one starts with the metric for a rotating M5-brane [33][7], Wick-rotated along the time direction and one spatial brane direction:

$$
\begin{aligned}
ds_{11}^2 &= H^{-1/3}\left[-dt^2 + d\mathbf{x}_{(4)}^2 + \frac{\Delta + a^2\sin^2\theta}{\Sigma}\left(dx_{11} - \frac{\mu a \cosh\delta \sin^2\theta}{r(\Delta + a^2\sin^2\theta)}d\varphi\right)^2\right] \\
&+ H^{2/3}\left[\Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\Omega_2^2\right], \quad (54)
\end{aligned}
$$

with $\Delta = r^2 - a^2 - \mu r^{-1}$ and $H = 1 + \frac{\mu \sinh^2\delta}{r\Sigma}$. Reduction along x_{11} results into a D4-brane that has grown a D6-brane with worldvolume $\mathbf{R}^{1,4} \times S^2$. By twisting the direction of the reduction, as described in Section 2, the conical singularities at $r = r_+$ can be cancelled out and the system be equilibrated.

In order to obtain the configuration for a D0-brane blown up into the D2-brane sphere, one simply performs a T-duality transformation along the directions of the D4. As a result, the D0-D2 configuration is delocalized in these four directions. The resulting string-frame metric is

$$
\begin{aligned}
ds^2 &= -H^{-1/2}\Lambda^{1/2}dt^2 + H^{1/2}\Lambda^{1/2}\left[\Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + r^2\cos^2\theta d\Omega_2^2 + \frac{\Delta\sin^2\theta}{\Lambda}d\varphi^2\right] \\
&+ H^{1/2}\Lambda^{-1/2}d\mathbf{x}_{(4)}^2, \quad (55)
\end{aligned}
$$

[7]A few typos in [33] were corrected in [34].

and dilaton $e^{\phi} = H^{3/4}\Lambda^{-1/4}$. The function Λ is as in (51), but now where r^{-4} appears, it must read r^{-1}. The system is equilibrated for

$$B = \frac{3r_+ + a}{2r_+(r_+ + a)\cosh\delta} \,. \tag{56}$$

5.3 The F1 and D0 in the D2 tube

This configuration has the same structure as the "supertube" discussed in [20] (see also [36]). Recall that, in the Dirac-Born-Infeld picture, such a supertube could be held in stable equilibrium at a finite radius of the tube, without the need of a RR external field to support the D2 tube against collapse.

The eleven-dimensional starting point is the rotating intersecting M2-M5 system. These intersect over a string. By Wick-rotating both the time and the space directions along this string, as well as the rotation parameter a, one finds

$$\begin{aligned} ds^2_{11} &= H_m^{-1/3}H_e^{-2/3}\left[-dt^2 + \frac{\Delta + a^2\sin^2\theta}{\Sigma}\left(dx_{11} - \frac{\mu a\cosh\delta_e\cosh\delta_m\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi\right)^2\right] \\ &+ H_m^{2/3}H_e^{-2/3}dz^2 + H_m^{-1/3}H_e^{1/3}d\mathbf{x}^2_{(4)} \\ &+ H_m^{2/3}H_e^{1/3}\left[\Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\psi\right] \,, \end{aligned} \tag{57}$$

where now $\Delta = r^2 - a^2 - \mu$, and

$$H_{e,m} = 1 + \frac{\mu\sinh^2\delta_{e,m}}{\Sigma} \,. \tag{58}$$

The expressions for the different components of the three-form potential can be obtained from [33]. We shall not need them here, though.

The indices e, m, refer to the electric M2 and magnetic M5, respectively. The M2 and M5 share the direction x_{11}, which is the one to be wrapped. In addition to this, the M2 extends also along z, while the M5 spans the four coordinates $\mathbf{x}_{(4)}$. Hence, reduction to type IIA theory will result into an F1 extended along z, and a D4 along $\mathbf{x}_{(4)}$. Both will be dissolved into a tubular D6 which spans $(z, \mathbf{x}_{(4)}, \psi)$. In order to achieve the configuration we seek, we T-dualize the four D4 directions. The metric, in string frame, is

$$\begin{aligned} ds^2 &= \left(\frac{\Delta + a^2\sin^2\theta}{\Sigma}\right)^{1/2}\left\{-H_m^{-1/2}H_e^{-1}dt^2 + H_m^{1/2}H_e^{-1}dz^2 + H_m^{-1/2}d\mathbf{x}^2_{(4)}\right. \\ &+ \left. H_m^{1/2}\left[\Sigma\left(\frac{dr^2}{\Delta} + d\theta^2\right) + \frac{\Sigma\Delta\sin^2\theta}{\Delta + a^2\sin^2\theta}d\varphi^2 + r^2\cos^2\theta d\psi\right]\right\} . \end{aligned} \tag{59}$$

An analysis of the geometry near the core of the D2 tube, along the lines of that in subsection 5.1, shows that near this core the solution reproduces the core of the bound state of an F1 and a D0 dissolved in the worldvolume of a flat D2.

Now we address the question of whether this configuration might be balanced in the absence of a fluxbrane. To this effect, we analyze the possible conical singularities on the fixed-point set of the vector ∂_φ, *i.e.,* the disk at $r = r_+ = \sqrt{\mu + a^2}$. Since a conformal rescaling of the metric coefficients g_{rr} and $g_{\varphi\varphi}$ does not affect the conical structure, it is easy to see that the result is the same as (15), namely,

$$\lim_{r \to r_+} \frac{2\pi}{\sqrt{g_{rr}}} \frac{d\sqrt{g_{\varphi\varphi}}}{dr} = \frac{2\pi\sqrt{\mu + a^2}}{a}\,, \tag{60}$$

which is independent of the D0 and F1 charges, and does not equal 2π for any non-trivial choice of parameters. The conical singularity remains. Disappointingly, the tube, by itself, cannot be in equilibrium (stable or unstable), whatever the value of the D0 and F1 charges.

6 Discussion

The conclusion of the analysis of this last configuration is puzzling, and it poses the following question: To what extent do the configurations we have obtained correspond to similar configurations, which have been studied earlier in the absence of self-gravity (closed string effects)?

Let us specify the terms for the comparison. A convenient way to study brane tubes and spheres, from the point of view of open strings, is by using the Dirac-Born-Infeld description of the worldvolume dynamics of a D-brane. In order to find the equilibrium configurations of these systems, one studies their static potential energy, for a given field strength and brane charge, as a function of the radius of the tube (or sphere). For an F-string of charge q_s, blown up into a tubular Dp-brane under the action of a RR $p+2$-form field of strength B, this potential was computed in [18], with the result

$$V_p(R) = \sqrt{R^{2p-2} + q_s^2} - \frac{B}{p} R^p\,, \tag{61}$$

in appropriate units. Tubes in equilibrium under the action of the external field have radii that correspond to the extrema of this potential. For $p = 2$, the shape of this potential is the following: $R = 0$ is a stable minimum, and if $B < |q_s|^{-1}$, there is a maximum at $R = \sqrt{B^{-2} - q_s^2}$. By contrast, for $p \geq 3$, the extremum at $R = 0$ becomes a local unstable maximum, and a minimum appears at a finite value of the radius R. For larger values of R

(and B less than a critical value) one finds again a (global) maximum, with a large value of V_p, *i.e.,* an unstable equilibrium configuration.

Essentially the same potential as (61) describes a Dq-brane dissolved in a tubular D$(q+2n)$-brane, after identifying $2n = p-1$ [18]. The minimum of this potential for the case of a D0 expanded into a spherical D2 (*i.e.,* the case $p = 3$ for (61)) corresponds to a dielectric configuration. These were studied in [19], where the existence of such configurations was also established from the point of view of matrix theory.

On the other hand, for the supertube, the potential energy, in the absence of any external field, is [20]

$$V(R) = \frac{1}{R}\sqrt{(q_s^2 + R^2)(q_0^2 + R^2)}\,, \tag{62}$$

where q_s and q_0 are the F-string and D0-brane charges, respectively. This potential possesses a unique extremum, a minimum at $R = \sqrt{|q_s q_0|}$.

Obviously, we have been unable to find, in our supergravity configurations, this minimum of the supertube potential. No such stable equilibrium configuration appears to be possible. This leads us to question whether the other configurations, namely the F1 in the Dp tube, and the D0 in the D2 sphere, can correspond to the configurations at the minima of the potentials. In this respect, notice that the Dirac-Born-Infeld potential (61) does not yield any stable minimum for the radius for the case of an F-string expanded into a D2 tube, so in this case it appears fairly clear that the gravitating configuration must correspond to this unstable point. Given these considerations, it may sound plausible that, also for $3 \le p \le 6$, the configurations we have built are associated to the tubes at the unstable maxima. Otherwise, one should expect to find, for $3 \le p \le 6$, two different equilibrium configurations, one stable and the other one unstable, for fixed values of the string charge, the external field, and, possibly, the number of tubular p-branes.

Similar considerations apply to the solution for a D0-brane blown up into a D2 sphere. In his study of dielectric branes, [19], Myers considered a regime in which one zooms in on the region between zero radius and the stable minimum. In this regime, the unstable maximum is pushed to a much larger value of the radius, and to a very high energy. Effectively, it disappears. It may be that the situation described in this paper is the opposite, in which only the unstable maximum is seen.

These observations, however, are not conclusive, and in order to reach a firmer verdict, a more detailed study of these configurations would be necessary. One possibility is to study the energetics of the supergravity solutions, possibly considering the off-shell configurations where the conical singularity is not eliminated. Such off-shell configurations would presumably correspond to points other than the extrema in the potentials above. Perhaps in this way one can determine whether the equilibrium configurations sit at a maximum or a minimum.

Another possibility, at first sight much more complicated, is to address directly the classical linearized stability of the configurations. As observed in [10], the instability of the tubular p-branes (without a net charge) is manifested by the existence of an unstable mode in the fluctuations around the Euclidean black holes [37][8]. Perhaps this argument can be modified so as to yield a proof of the instability of all the configurations described in this paper. If this were the case, a description of the dielectric effect that accounts for the self-gravity of the branes would remain an elusive problem.

At any rate, it appears clear that a detailed analysis of the properties of these solutions is required. In this paper we have applied and extended the techniques of [10], and managed to construct a variety of solutions describing exact tubular branes in string and M-theory. Our focus has been mostly on the construction of the solutions. Further study of their properties is left for future work.

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